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MATHEMATICAL MODEL OF LEAST COST PLANNING OF REGIONAL ENERGY SUPPLY

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A number of simple mathematical models for least cost planning of energy supply are considered. Optimum plans have been found and ranking of energy sources is suggested.

Introduction. At present, regional least cost planning has been widely recognized as the most effective method in the system of long-term forecasting of energy supply [1]. In the general case, least cost planning is a process of development and implementation of the strategy of using a resource or a set of resources that are able to provide for meeting the need for energy supply in a particular region at the lowest overall cost, including environment protection, competitiveness of new energy sources compared with those traditional for this power system, and some other situational considerations.

The development of planning starts from analysis of demographic trends in the region, the potential of economic development, and the present need for energy services, and on this basis a forecast of the ranges of energy consumption for the near future is made. Then, traditional and potential sources are compared to determine their reliability and cost in meeting the energy needs in the region. In this connection, development of mathematical approaches suitable for planning, which could be used to obtain optimum estimates of the planned power output for the various sources, is important. Dynamic, linear, and nonlinear programming methods compose the theoretical basis of these approaches [2–4].

In the present work some simple mathematical models of least cost planning of power supply are considered, optimum plans are found, and ranking of sources is suggested that reflects particular aspects of their economic estimation. In particular, depending on the structure of the cost function of the energy production, a procedure of differentiation of the sources in the level of their "cost" is suggested, and prospects of using the various sources are estimated with regard to expansion or reduction of the output. Problems of estimation of stability of the optimum plans under variations in the planned power output are considered.

1. Mathematical Statement of the Problem. Let us consider n sources with the power output E_i for the i-th source. The quantity E_i is expressed in the form

$$E_i = E_{i0} + \Delta E_i , \quad i = \overline{1, n} . \tag{1}$$

It should be noted that the basic level for particular sources (power plants under construction, promising energy-effective technologies, etc.) can be zero and the planned power increment can be negative (reduction of the output at environmentally harmful plants, reduction of power import, etc.). Next, we will consider the expected cost φ_i of unit power production by the i-th source as a function of the planned power output. In the general case we have

$$\varphi_i = \varphi_i (E) , \quad E = (E_1 , \dots , E_n) , \quad i = \overline{1, n} . \tag{2}$$

Functions (2) will have to be expressed analytically. It should be noted that costs (2) depend on some factors, among which there are both difficultly predictable and quite definite ones, and therefore description of the cost functions is complicated. The first step to simplify the description is to assume that the cost of unit power

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produced by the i-th source is independent of the power outputs by the other sources j, $j \neq i$. Thus, it will be assumed that $\varphi_i(E) = \varphi_i(E_i)$ and the function φ_i will be expanded in a Taylor series in a neighborhood of the basic level E_{i0}. We have

$$\varphi_i (\Delta E_i) = \varphi_{i0} + \varphi_{i1} \Delta E_i + \varphi_{i2} (\Delta E_i)^2 + \ldots , \quad i = \overline{1, n} , \tag{3}$$

where φ_{i0} is the cost of unit power production E_{i0} at the basic level .

The simplest models for the cost function follow from Eq. (3):

$$\varphi_i (\Delta E_i) = \varphi_{i0} , \tag{4}$$

$$\varphi_i (\Delta E_i) = \varphi_{i0} + \varphi_{i1} \Delta E_i , \tag{5}$$

$$i = \overline{1, n} ,$$

which are applicable for the practical case of sufficiently large planned power increments E_i. Apart from Eqs. (4) and (5), fractional power representation of function (3) is used in economic literature:

$$\varphi_i (\Delta E_i) = k_i (E_i)^{\nu_i} \equiv k_i (E_{i0} + \Delta E_i)^{\nu_i} , \quad i = \overline{1, n} . \tag{6}$$

where $k_i = (E_{i0})^{-\nu_i} \varphi_{i0}$ and ν_i is the elasticity factor of the function. If $-1 < \nu_i < 0$, functions (6) model a typical situation where cost of unit power production decreases as the power output rises and the total cost $\varphi_i(E_i)E_i$ increases simultaneously.

The fractional polynomial approximation of the functions φ_i (Padé approximant) is also rather flexible:

$$\varphi_i (\Delta E_i) = \frac{\varphi_{i0} + \alpha_1 \Delta E_i + \ldots + \alpha_p (\Delta E_i)^p}{1 + \beta_1 \Delta E_i + \ldots + \beta_k (\Delta E_i)^k} . \tag{7}$$

We will formulate the problem of minimizing the total cost of the power production in the following way. The cost functions for power production by the i-th power source will be formed as

$$\omega_i (\Delta E_i) = \varphi_i (\Delta E_i) (E_0 + \Delta E_i) , \quad i = \overline{1, n} , \tag{8}$$

and the total cost function will be considered:

$$\omega (\Delta E) = \sum_{i=1}^{n} \omega_i (\Delta E_i) . \tag{9}$$

The objective of optimization is to determine the optimum plan $\Delta E^* = (\Delta E_1^*, \ldots, \Delta E_n^*)$ providing the minimum function $\omega(\Delta E)$

$$\omega (\Delta E^*) = \min_{\Delta E} \omega (\Delta E) , \tag{10}$$

with the additional limitations

$$\sum_{i=1}^{n} \Delta E_i = Q , \tag{11}$$

$$\Delta E_{i\,\min} \leq \Delta E_i \leq \Delta E_{i\,\max} , \quad i = \overline{1, n} . \tag{12}$$

Relation (11) ensures the planned power increment Q, and inequalities (12) describe admissible ranges of change of the power output. The quantities $\Delta E_{i\,\min}$ and $\Delta E_{i\,\max}$ depend on many factors, among which we can mention socio-economic, technological, etc. Compatibility of limitations (11) and (12) is equivalent to satisfying the condition

$$\sum_{i=1}^{n} \Delta E_{i\,\min} \le Q \le \sum_{i=1}^{n} \Delta E_{i\,\max}. \tag{13}$$

It should be noted that the planned increment can be both positive (expansion of the output) and zero (preservation of the total achieved level) or negative (reduction of the output). In each of the cases enumerated, minimization problem (10)-(12) has meaning. For example, if $Q \le 0$, the optimum plan describes the optimum redistribution and reduction of the existing output.

In the general case the optimum plan can be found with the use of numerical algorithms employed in the method of dynamic programming well known in the theory of optimum processes [2]. However, in standard problems solved by this method there are no upper bounds set on the functions, and therefore in Appendix I the method of dynamic programming is described with account for such limitations.

2. Examples of Explicit Solution and Analysis of the Optimization Problem. Along with numerical realization, in the general situation for analysis and qualitative investigation of least cost planning, cases of explicit solution of optimization problem (6) are of great interest. In particular, the problem of linear programming

$$\sum_{i=1}^{n} \varphi_{i0}\Delta E_i \to \min, \quad \sum_{i=1}^{n} \Delta E_i = Q, \quad \Delta E_i \in [\Delta E_{i\,\min}, \quad \Delta E_{i\,\max}], \quad i = \overline{1, n}, \tag{14}$$

having an explicit solution, follows from general problem (10)-(12) with choice of the cost function in the form of Eq. (4).

For a description of the solution of problem (13), the sources will be arranged in the order of increasing costs of unit power production in the basic outputs:

$$\varphi_{10} \le \varphi_{20} \le \ldots \le \varphi_{n0}. \tag{15}$$

Then, the source j satisfying the conditions

$$\sum_{i=1}^{j-1} \Delta E_{i\,\max} + \sum_{i=j}^{n} \Delta E_{i\,\min} < Q, \quad \sum_{i=1}^{j} \Delta E_{i\,\max} + \sum_{i=j+1}^{n} \Delta E_{i\,\min} \ge Q. \tag{16}$$

will be determined.

Then, the optimum plan has the form

$$\Delta E_i^* = \begin{cases} \Delta E_{i\,\max}, & \text{if} \quad i \in \{1, 2, \ldots, j-1\}, \\ Q - \sum_{i=1}^{j-1} \Delta E_{i\,\max} - \sum_{i=j+1}^{n} \Delta E_{i\,\min}, & \text{if} \quad i = j, \\ \Delta E_{i\,\min}, & \text{if} \quad i \in \{j+1, \ldots, n\}, \quad i = \overline{1, n}. \end{cases} \tag{17}$$

The proof of optimality of plan (17) is given in Appendix II.

It should be noted that in this very simple case the structure of optimum plan (17) is quite consistent with the intuitive idea of the optimum decision: the power output from "expensive" power sources is reduced as much as possible, while the power output from "cheap" sources is increased as much as possible. Ranking of the sources as

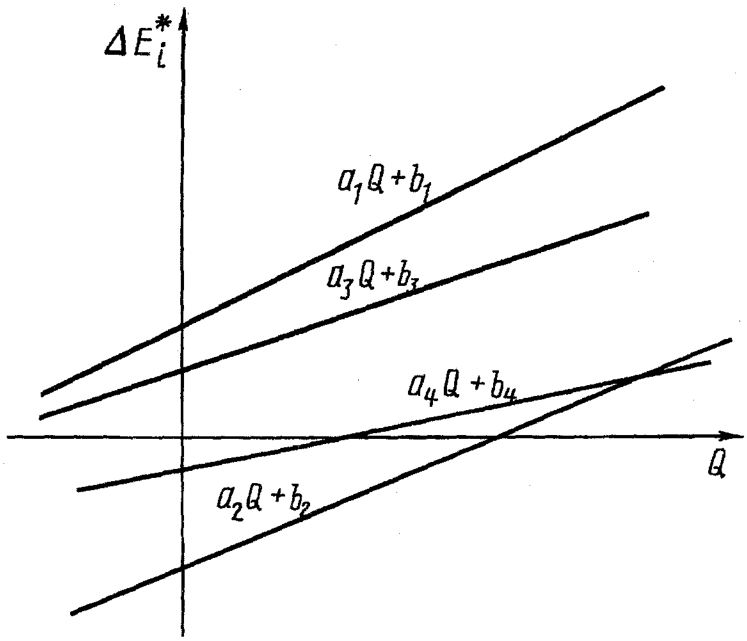

Fig. 1. Graphical representation of the dependence of the optimum plan on the planned power increment.

expensive and cheap is determined by inequalities (16), which identify the boundary source j with an intermediate structure of the production.

The stability of the ranking and the optimum plan (17) under variations in the planned output Q of power increment depends on the potentialities of the boundary source j. In general situations slight variations in Q are compensated by changes in the power output by the boundary source j. With substantial changes in Q the boundary source is displaced and structural rearrangements occur in the sources next to the boundary one.

Using the theory of Lagrange multipliers [3], one can easily obtain an explicit solution of optimization problem (10) also in the case where the cost functions have the form of Eq. (5), $\varphi_{i1} > 0$, $i = \overline{1, n}$, and the variables ΔE_i have a lower bound set by the natural limit $-E_{i0}$, i.e., where the inequalities

$$E_{i0} + \Delta E_i \geq 0, \quad i = \overline{1, n}. \tag{18}$$

are satisfied.

Moreover, it will be assumed that after freezing of the production, the cost of "production" of unit power is zero, and then $\varphi_i(-E_{i0}) = 0$, $i = \overline{1, n}$.

With the above limitations the optimum plan is calculated from the formulas

$$\Delta E_j^* = (\varphi_{j1} B)^{-1} \left(Q + \sum_{i=1}^{n} E_{i0} \right) - E_{j0}, \quad j = \overline{1, n}, \tag{19}$$

where $B = \sum_{i=1}^{n} \varphi_{i1}^{-1}$.

One more explicit solution of problem (10) will be given, where the cost functions are chosen in the form of Eq. (9), the elasticity factors are equal for all the sources ($\nu_i = \nu$, $i = \overline{1, n}$), and inequalities (18) are satisfied. Then,

$$\Delta E_j^* = k_j^{\frac{1}{\nu}} D^{-1} \left(Q + \sum_{i=1}^{n} E_{i0} \right) - E_{j0}, \quad i = \overline{1, n}, \tag{20}$$

where $D = \sum_{i=1}^{n} k_i^{1/\nu}$.

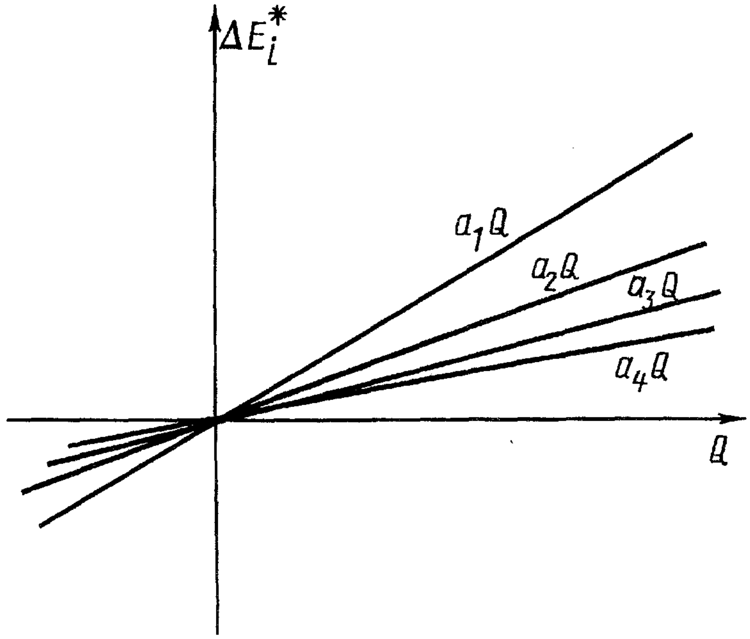

Fig. 2. Plot of the optimum plan versus Q in the case of a balanced structure of the basic power production.

Unfortunately, formulas (19) and (20) are not as obvious as optimum plan (17). However, optimum plans (19) and (20) have the remarkable property that they are linear functions of the planned volume of power increment Q. Indeed, relations (10) and (20) can be reduced to the formula

$$\Delta E_j^* = a_j Q + b_j\,, \quad j = \overline{1, n}\,, \tag{21}$$

where $a_j = 1/\varphi_{j1}B$, $b_j = a_j \sum_{i=1}^{n} E_{i0} - E_j$ for formula (10) and $a_j = k_j^{1/\nu} D$, $b_j = a_j \sum_{i=1}^{n} E_{i0} - E_j$ for formula (20). The coefficients a_j and b_j clearly satisfy the relations

$$\sum_{i=1}^{n} a_i = 1\,, \quad a_i < 1\,, \quad i = \overline{1, n}\,; \quad \sum_{i=1}^{n} b_i = 0\,. \tag{22}$$

The linearity of functions (21) can be used conveniently for the ranking of sources, reflecting their dynamism and prospects in the case of expansion (or, on the other hand, reduction) of power output. Evidently, this ranking is determined by the chain of inequalities

$$1 > a_1 \geq a_2 \geq \ldots \geq a_n > 0\,. \tag{23}$$

A graphical representation of functions (21) also gives an obvious idea of the dynamism of particular sources (Fig. 1). It should be noted that the simplest situation occurs when the structure of the basic power production has already been balanced and, consequently, is optimum. In this case the basic levels E_{i0} of the sources satisfy the system of equations

$$b_i\,(E_{i0}) = 0\,, \quad i = \overline{1, n}\,, \tag{24}$$

and optimum plan (21) is described by the relation $\Delta E_j^* = a_j Q_j$, $j = \overline{1, n}$ (see Fig. 2).

APPENDIX I

The solution of optimization problems by the method of dynamic programming is based on construction of the Bellman function. For problem (10)-(12) the Bellman function $B_s(y)$ is determined as follows:

$$B_s(y) = \min_{\Delta E} \sum_{i=1}^{s} \omega_i(\Delta E_i), \quad s \in \{1, \ldots, n\}. \tag{A.I.1}$$

In this case the components of the vector ΔE should a satisfy the conditions

$$\Delta E_i \in [\Delta E_{i\,\min}, \Delta E_{i\,\max}], \quad i = \overline{1, n}, \quad \sum_{i=1}^{s} \Delta E_i = y \in Y_s, \tag{A.I.2}$$

$$Y_s = \left[\sum_{i=1}^{s} \Delta E_{i\,\min}, \ \sum_{i=1}^{s} \Delta E_{i\,\max} \right] \cap \left[Q - \sum_{i=s+1}^{n} \Delta E_{i\,\max}, \ Q - \sum_{i=s+1}^{n} \Delta E_{i\,\min} \right]. \tag{A.I.3}$$

It follows from (A.I.1), (A.I.2), and the optimality principle [2] that the function $B_s(y)$ is a solution of the Bellman equation

$$B_{s+1}(y) = \min_{\Delta E_{s+1} \in \theta_{s+1}(y)} [\omega_{s+1}(\Delta E_{s+1}) + B_s(y - \Delta E_{s+1})], \tag{A. I. 4}$$

where the set $\theta_{s+1}(y)$ has the form

$$\theta_{s+1}(y) = \left\{ \Delta E_{s+1} : \ \Delta E_{s+1} \in [\Delta E_{s+1\,\min}, \Delta E_{s+1\,\max}] \cap (y - Y_s) \right\}, \quad y \in Y_{s+1}.$$

Moreover, the following equality holds:

$$B_1(y) = \omega_1(y), \quad y \in Y_1, \tag{A. I. 5}$$

which should be considered as the initial condition for equation (A.I.4).

Thus, using Eqs. (A.I.5) and (A.I.4) one can calculate successively the functions $B_1(y)$, $B_2(y)$ ... . The functions $\Delta E_s = \alpha_s(y)$, $s = \overline{1, n}$ providing a maximum to (A.I.1) are determined concurrently. The optimum plan is also calculated successively, but in "inverse time" with the aid of the recurrence relations

$$\Delta E_k^* = \alpha_k \left(Q - \sum_{i=k+1}^{n} \Delta E_i^* \right), \quad \Delta E_n^* = \alpha_n(Q).$$

The brief description of the dynamic programming procedure given here differs from the conventional description by a more thorough description of the set Y_s.

APPENDIX II

The proof of optimality of plan (17) is based on the use of a well-known theorem of linear programming [3] that states that admissible plans* are optimum if and only if the corresponding values of optimizable functionals coincide.

For simplification of the subsequent calculations we make the substitution of variables $x_i = d\Delta E_i - \Delta E_{i\,\min}$ and use the following notation: t is the transposition symbol, $x = [x_1, \ldots, x_n]^t$; $h_i = \Delta E_{i\,\max} - E_{i\,\min}$; $h = [h_1, \ldots, h_n]^t$; $\varphi_0 = [\varphi_{10}, \ldots, \varphi_{n0}]$; $d = [1, \ldots, 1]$; $q = Q - \sum_{i=1}^{n} \Delta E_{i\,\min}$; $\langle \cdot, \cdot \rangle$ is the scalar product in the Euclidean space $\mathbf{R}^n$.

Problem (14) is equivalent to the following one:

$$\langle \varphi_0, x \rangle \to \min, \quad 0 \le x \le h, \quad \langle d, x \rangle = q. \tag{A. II. 1}$$

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* It should be noted that the plan is admissible if it satisfies the limitations for the problem of linear programming.

The problem dual to (A.II.1) has the following form:

$$qy_{n+1} - \langle h, x \rangle \to \max (y_{n+1} \in \mathbf{R}, y \in \mathbf{R}^n), \quad d^{\mathrm{T}} y_{n+1} - y \le \varphi_0, \quad y \ge 0. \tag{A.II.2}$$

Admissibility of the plan for problem (A.II.1)

$$x_i^* = \begin{cases} h_i, & i = \overline{1, j-1}, \\ q - \sum\limits_{i=1}^{j-1} h_i, & i = j, \\ 0, & i = \overline{j+1, n}, \quad i \in \{1, 2, \dots, n\}, \end{cases} \tag{A.II.3}$$

and of the plan for problem (A.II.2)

$$y_i^* = \begin{cases} \varphi_{j0} - \varphi_{i0}, & i = \overline{1, j-1}, \\ 0, & i = \overline{j, n}, \\ \varphi_{j0}, & i = n+1, \quad i \in \{1, 2, \dots, n+1\}, \end{cases} \tag{A.II.4}$$

is verified directly. Here j satisfies the inequalities $\sum\limits_{i=1}^{j-1} h_i < q$ and $\sum\limits_{i=1}^{i} h_i \ge q$. On the other hand, it is clear that $< \varphi_0, x^* > = qy_{n+1}^* - < h, y^* >$. Therefore, according to the above theorem, plans (A.II.3) and (A.II.4) are optimal. Hence, optimality of plan (17) follows immediately.

NOTATION

E_i, power output of the i-th source; E_{i0}, basic level of power production; ΔE_i, planned power increment; φ_i, cost of unit power production; ν_i, elasticity factor of the function φ_i; Q, total planned power increment; $B_s(y)$, Bellman function.

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